Matrix and Variant Transformations Simulate Statistical Canonical Ensembles from Fock to Poissonian States of Random Sequences

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Abstract

From a quantum statistical viewpoint, four typical quantum states are Fock, Sub-Poissonian, Poissonian and Super-Poissonian states. Quantum interactions are among Fock and Poissonian states. Using quantum statistics, model and simulation, this paper proposes two models: matrix and variant transformations: 1. MT Matrix Transformation – eigenvalue states; 2. VT Variant Transformation – invariant states to analyze three random sequences: 1) random; 2) conditional random in a constant; 3) periodic pattern. Four procedures are proposed. Fast Fourier Transformation FFT is applied as one of MT schemes and two invariant scheme of VT schemes are applied, three random sequences are used in M segments, and each segment has a length m to generate a measuring sequence. Shifting operations are applied on each random sequence to create m+1 spectrum distributions. Better than FFT, VT can identify Fock, Sub-Poissonian, Poissonian states in random analysis to distinguish three random sequences as three levels of statistical ensembles: Micro-canonical, Canonical, and Grand-Canonical ensembles. Applying two transformations, quantum statistics, model and simulation of modern quantum theory and applications can be explored. Copyright © VBRI Press.

Keywords: Fock, sub-poissonian, poissonian, super-poissonian, matrix transformation, variant transformation, canonical ensembles.

Introduction

In quantum optics, quantum statistics and photon statistics play key roles. From a spectrum analysis viewpoint, quantum statistics are significantly different from classical random signal sequences.

Quantum states

Using photon counting technology, it is possible for photonic signal sequences to obtain statistical properties of photons. From a quantum state viewpoint, quantum photonic statistics correspond to three quantum states [1] shown in **Fig. 1**:

- Super-Poissonian: Amplitude-squeezed state
- Poissonian: Coherent state
- Sub-Poissonian: Phase-squeezed state

Two states: Super-Poissonian and Poissonian states correspond to semi-classic wave distributions as coherent states. However, sub-Poissonian states are linked with particle-based quantum interactions with quantum squeezed coherent effects.

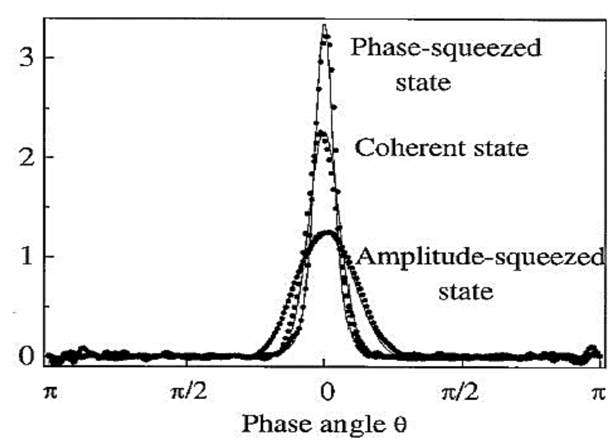

Fig. 1. Three quantum states: Super-Poissonian (Amplitude-squeezed state), Poissonian (Coherent state), Sub-Poissonian: (Phase-squeezed state).

Stationary/non-stationary random processes

In quantum measurement schemes, stationary/non-stationary random properties play a key role to identify classic/semi-classic or quantum interactions.

- Classical coherent states: if angular variables are changed in phase spaces, quantum states keep in Poissonian states to make statistical parameters with invariant properties – stationary processes
- Quantum squeezed coherent states: if angular variables are changed in phase spaces, quantum states will be changed from sub-Poissonian to Poissonian states to make statistical parameters with significantly variant properties – non-stationary processes.

Questions

Using models and technologies of photonic statistics and quantum optics such as coherent Lasers, optical fibers, it is convenient to model and simulate classic coherent states and quantum squeezed coherent states. Whether it is possible to use discrete framework to make proper simulation, following two questions need to be asked:

1. Using 0-1 random sequences, is it feasible to generate variations from sub-Poissonian states to Poissonian states?
2. Under discrete statistical processes, does it be possible to handle stationary/non-stationary random processes?

Matrix transformation and eigenvalue spectra

Spectral analysis considers the problem of determining the spectral content (i.e., the distribution of power over frequency) of a time series from a finite set of measurements, by means of either nonparametric or parametric techniques. The history of spectral analysis as an established discipline started more than a century ago with the work by Schuster [**2**] on detecting cyclic behavior in time series.

Signal analysis and processing

In a modern digital environment, spectrum analysis plays a key role in signal processing that is a subfield of mathematics and information sciences to concern the analysis, synthesis, and modification of signals such as sound, images, and biological measurements. For example, signal processing techniques are used to improve signal transmission, storage efficiency, and subjective quality, and to detect components of interest in a measured signal. This contains multiple subjects such as Matrix Theory [**3**], Non-continuous Orthogonal Functions [**4**], Probability [**5**], Transform Theory [**6**], Time Series [**7**], Linear Algebra [**8**], Time-Frequency Analysis [**9**], Stochastic Processes [**10-12**], Spectral Estimation [**13, 14**], Statistical Signal Analysis [**15**], Non-Linear Spectral Analysis [**16**], Matrix Analysis [**17**] etc.

Variant construction

In relation to discrete spectra, variant construction is an emerging approach to use multiple invariants to analyze random sequences in various applications such as stationary randomness of quantum cryptographic sequences [**18**], Chaotic random sequences [**19**] and variant construction [**20**].

Results in this paper

In this paper, two transformations: MT Matrix Transformation and VT Variant Transformation on discrete sample sequences are proposed on three 0-1 random sequences to generate various distributions under multiple parameter conditions. Sample cases are provided to show three quantum states: Fock, sub-Poissonian and Poissonian states under discrete framework.

Simulation model

For a discrete sequence with N elements, it is convenient to be separated as segments in multiple pieces and each segment with m variables to support various applications.

Two transformations on statistics

Both two transformations use m variables as input to generate either m eigenvalues or two invariant measures as output. Five components are identified: Input, Transformation, Output, Distribution, Map.

Key components

Under MT and VT, it is convenient to use three random sequences on multiple segments and each segment with a constant length m to generate a series of spectra under various conditions. A set of shifting operations can be applied on a random sequence to generate m+1 sets of spectra. This type of shifting results on phase spaces can be created rich comparison effects on the three random sequences. Four procedures of Case A to Case D are described.

Case A: MT on m variables

Input: m 0-1 variables
Transformation: An m×m Matrix
Output: m eigenvalues
Distribution: Two histograms are distinguished by eigenvalues
Map: Two 1D maps on real and imaginary parts

Case B: MT on m*M variables

Input: m*M 0-1 variables
Transformation: m → An m×m Matrix; r → initial shifting position
Output: m*M eigenvalues
Distribution: Two histograms are distinguished by eigenvalues
Map: Two 1D maps on real and imaginary parts

Case C: VT on m variables

Input: m 0-1 variables
Transformation: Two equations: p 1 and q 01 numbers
Output: A pair of {p, q} measures
Distribution: Two histograms are distinguished by {p, q}
Map: A single point on two histograms

Case D: VT on m*M variables

Input: m*M 0-1 variables
Transformation: m → segment length; r → initial shifting position
Output: M pairs of {p, q} measures
Distribution: Two histograms are distinguished by M pairs of measures
Map: Two 1D maps on 1DP and 1DQ distributions

From a measuring viewpoint, both Case A and Case C provide a single pass of measurements. There are maximal m positions distinguished on Case A in each map. However, Case C contains only a single point on each map. In addition, both Case B and Case D take M passes of measurements. Both cases are created specific distributions on a pair of maps under different conditions.

Three selected random sequences

The Original Random Sequence ORS 100MB is collected from the ANU quantum random number server [**21**] as a quantum random resource. Using this original sequence, three sub-sequences on each 0.8MB length are selected by various variation properties:

1. A sub random sequence from ORS;
2. A relative random sequence filtered by a micro canonical ensemble in a constant restriction from ORS;
3. A periodic sequence of special pattern selected from ORS.

Visual results

Using proposed schemes, a 1D FFT is used as one of MT schemes on Case B, and two invariants are extracted as one of VT schemes on Case D. Three random sequences are applied to generate various maps respectively. Three groups of various results on nine maps are shown in **Fig. 2** (a1-a3)-(c1-c3) on m=128, M=6400 respectively.

Undertaken shift operations, five sets of results are generated and results of twelve groups on 60 maps are shown in **Fig. 3** (a1-a5)-(l1-l5) on m = 128, M = 6400, r = {0, 1, 20, 40, 64} respectively.

Result analysis

Analysis on Fig. 2

From three groups of results shown in **Fig. 2** (a1)-(c1), the most distributions of maps (a1) and (b1) are similar, they are complete different from map (c1). However, enlarged real histograms on maps (a2), (b2) and (c2), significant different distributions are observed in their middle parts.

Different from FFT maps, maps (a3)-(c3) are generated by VT schemes, 1DP map has a Poissonian distribution and 1DQ map has a sub-Poissonian distribution in map (a3). However, both 1DP and 1DQ in (b3) and (c3) maps have only a single spectrum in Fock states.

Analysis on Fig. 3

Special stationary and non-stationary properties are observed from shifting initial conditions to relevant input sequences. From twelve groups of results in **Fig. 3** (a?) - (l?), five distinct shifting numbers are selected and their corresponding maps are illustrated.

Stationary properties are observed in (a1)-(a5) with similar properties in (b1)-(b5); main parts of distributions in (c1)-(c5) are showing stationary properties, however, their enlarged parts of distributions in (d1)-(d5) have partial non-stationary properties.

There are similar distributions in (e1)-(e5) with stationary properties; however, significant differences are shown in enlarged distributions on (f1)-(f5) respectively.

Similar to distributions in (a1)-(a5) and (b1)-(b5), each group of four maps on either 1DP maps (g1)-(g5) or 1DQ maps (h1)-(h5) are shown in stationary properties under various shifting operations in VT schemes.

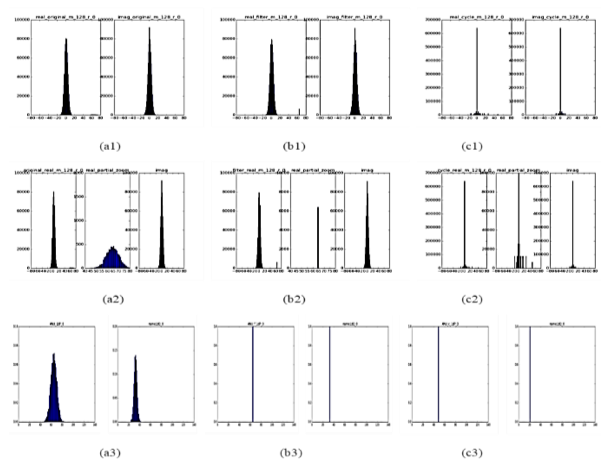

Fig. 2. Three sequences: {1), 2), 3)} under two schemes: {FFT, VT}, m=128, M=6400; (a1) FFT on 1); (b1) FFT on 2); (c1) FFT on 3); (a2) enlarged map on FFT of 1); (b2) enlarged map on FFT of 2); (c2) enlarged map on FFT of 3); (a3) VT 1DP and 1DQ on 1); (b3) VT 1DP and 1DQ on 2); (c3) VT 1DP and 1DQ on 3).

Different from distributions in (c1)-(c5) and (d1)-(d5), non-stationary properties can be significantly identified in each group of five maps on either 1DP maps (i1)-(i5) or 1DQ maps (j1)-(j5) shown in non-stationary properties under shifting operations in VT schemes. The distributions are sharply changed from Fork states to sub-Poissonian and Poissonian states according to various shifting parameters.

Different from distributions in (e1)-(e5) and (f1)-(f5) with enlarged differences, stationary properties can be significantly identified in each group of five maps on either 1DP maps (k1)-(k5) or 1DQ maps (l1)-(l5) with only a Fock state with a constant measurement.

Differences on two schemes

From listed results, it is interesting to notice that VT 1DP maps have corresponding distributions on FFT real distributions in partial regions. A VT 1DP map is a distribution to the measure p, significantly different from FFT with m eigenvalues in representation.

In general, it is hard to apply statistical mechanism to distinguish sequences 1) and 2) using a FFT scheme; five pairs of (a1) -(a5) and (c1) -(c5) have most parts in similar distributions within 1% parts shown in minor differences. Only two groups of sequences: {1), 2)} and 3) can be identified using the FFT scheme.

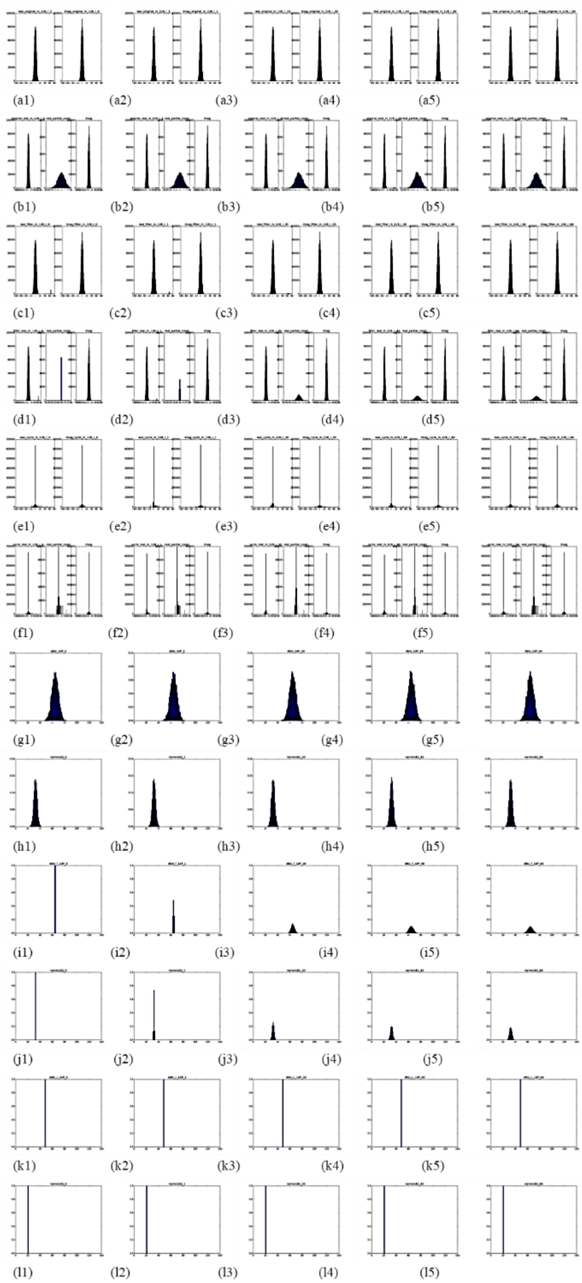

Fig. 3. Twelve groups of results on three sequences: {1), 2), 3)}, m=128, r={0,1,20,40,64}, M=6400; Maps (*1) r=0, (*2) r=1, (*3) r=20, (*4) r=40, (*5) r=64; * ∈ {a,…,l}, ? ∈ {1,2,3,4,5}; (a?) FFT maps on 1); (b?) enlarged FFT maps on 1); (c?) FFT maps on 2); (d?) enlarged FFT maps on 2); (e?) FFT maps on 3); (f?) enlarged FFT maps on 3); (g?) VT 1DP maps on 1); (h?) VT 1DQ maps on 1); (i?) VT 1DP maps on 2); (j?) VT 1DQ maps on 2); (k?) VT 1DP maps on 3); (l?) VT 1DQ maps on 3).

However, a VT scheme can directly identify all three random sequences in 1DP and 1DQ maps respectively. In addition, shifting operators describe dynamic properties of statistical processes in details. Using statistical ensembles, various variations can be identified.

- (g1)-(g5) and (h1)-(h5) of VT are shown in stationary random properties on sequence 1), global invariants in grand-canonical ensembles;
- (i1)-(i5) and (j1)-(j5) of VT are shown in non-stationary random properties on sequence 2), local invariants from micro-canonical ensembles to canonical ensembles;
- (k1)-(k5) and (l1)-(l5) of VT are shown in constant stationary properties on sequence 3), shifting invariants in periodic condition as Fock states.

In general, three distinct sequences: {1), 2), 3)} are naturally distinguished by the VT scheme to use stationary / non-stationary and random / periodic properties on three quantum states of statistical ensembles to identify various statistical processes respectively.

Conclusion

In this paper, two schemes: matrix and variant transformations are applied to discrete sequences. Both m eigenvalues and two invariants are extracted, statistical distributions are applied to organize multiple complex eigenvalue and invariant sequences as two 1D maps. Shifting operations are used to check stationary / non-stationary properties and random / periodic features for multiple segmented sequences.

Using three random sequences and two transformations: FFT and VT under statistical distributions, a set of testing results are compared. From result analysis, classical FFT scheme can classify three random sequences into two groups. However, the VT scheme has advances to identify three sequences as three groups. These visual maps provide initial experiment results using statistical ensembles in clear visual effects to link with Matrix Transformation and Variant Transformation. It is interesting to explore higher dimensional distributions based on multiple invariants in statistical ensembles, future explorations on quantum statistical theories and applications are required.

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Author's contributions

Jeffrey Zheng provides theoretical models and methods on matrix and variant transformations. Yamin Luo and Xin Zhang generate multiple statistical maps. Chris Zheng made confirmation for the testing.

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