Generation of Entanglement in Spin States of Rydberg Atoms by Chirped Optical Pulses

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Abstract

Quantum entanglement is a crucial resource in many quantum information and quantum communication tasks. In this work, we present a quantum control methodology to create entangled states of two basic classes, the W and the GHZ. A chain of $^{87}$Rb atoms in an optical lattice is considered interacting with laser pulses to induce two-photon excitations to Rydberg states having a specific magnetic quantum number. Generation of the W and GHZ three-atomic states is demonstrated via the mechanism of the two-photon adiabatic passage in collective states implying the overlapping chirped pulses and the interplay of the Rabi frequency with the one-photon detuning and the strength of the Rydberg-Rydberg interactions. Copyright © VBRI Press.

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Introduction

Rydberg atoms in an optical trap are viable systems to study collective electronic properties in many-body physics. The most prominent features of Rydberg atoms are strong long-range interatomic interactions and an extraordinary long lifetime of the Rydberg states [1, 2]. This makes Rydberg atoms an ideal platform to simulate interacting spin systems to understand and to control quantum many-body properties, e.g., quantum magnetism, quantum coherence and quantum entanglement. Quantum entanglement is a crucial resource in many quantum information and quantum communication tasks [3], examples include quantum dense coding [1], quantum key distribution [2], quantum cryptography [4], etc. Even though the entanglement of two atoms is well understood, the entanglement involving larger number of atoms, the multipartite entanglement, is far from being clear and is the subject of intense investigations. There are three typical classes of multipartite entangled states: the Greenberger-Horne-Zeilinger (GHZ) state [5], the W state [6], and the cluster state [7]. GHZ states are the best quantum channels for teleportation [8] and quantum key distribution [9], while W states are required for, e.g., the secure quantum communication [10, 11], and cluster states are basic resources for measurement-based quantum computation [12]. These classes of multipartite entangled states cannot be converted into each other with local operations and classical communication [13]. Therefore, it is important to learn to create entangled states of different classes on demand. In this work, we present a quantum control methodology to create entangled states of two basic classes, the W and the GHZ. *Alkali $^{87}$Rb atoms in an optical lattice interacting with laser pulses are used as a platform to demonstrate the quantum control methodology of two-photon excitations to entangled spin states of Rydberg atoms having entanglement properties on demand. These systems may be used as quantum sensors and as a source of entangled photons.

Theoretical

A schematic of magnetic sublevels - the spin states - involved in population dynamics is shown in Fig. 1a, chosen based on the following considerations for a correlation of the magnetic field strength and the relevant spin states involved in the excitation. The magnetic field strength must be sufficient for the Zeeman splitting to exceed the many-body state energy shift due to Rydberg blockade. At the same time, the Zeeman shift has to be within a single hyperfine splitting to avoid overlap of different hyperfine states. Therefore, for a few tens of MHz of the Rydberg-Rydberg interaction, a magnetic field of up to 100 G is sufficient. Two $\sigma^+_1,\sigma^+_2$ circularly polarized pulses having carrier frequencies $\omega_0(t)$ and $\omega_2(t)$ perform the two-photon transition within a single atom from state $|5S_{1/2},F=1,m_f=0\rangle$ to $|5P_{3/2},F=1,m_f=1\rangle$ to $|4D_{3/2},m_f=3/2,m_l=1/2\rangle$, forming a three-level ladder system. Selectivity of excitation of the spin states split by the Zeeman effect by tens of MHz is achieved by circularly polarized $\mu$s pulses, the magnetic field strength and the
MHz two-photon detuning. The ultracold atomic gas is modeled by an ensemble of three-level systems coupled by van der Waals interactions of states $|3\rangle$, shown in Fig. 1b.

**Results and discussion**

Generation of the W $|\Psi_W^3\rangle = \frac{1}{\sqrt{3}} (|133\rangle + |313\rangle + |331\rangle)$ and GHZ $|\Psi_{GHZ}^3\rangle = \frac{1}{\sqrt{2}} (|111\rangle + |333\rangle)$ three-atomic states is demonstrated via the mechanism of the two-photon adiabatic passage within the collective states performed by two, equally chirped pulses with overlapping respective Rabi frequencies [14]. Both classes of entangled states, the W and GHZ, are controllably generated by choosing the Rydberg-Rydberg interaction and Rabi frequency values ratio less than one and sweeping through the $\Delta$ values. The population of the states of the end of the pulse duration, shown in Fig. 2a, are then compared to find when the population between the required states is equally distributed.

The mechanism of the adiabatic passage leading to generation of the entangled GHZ three-atomic state is manifested in the time-dependence of the populations of the collective ground $|111\rangle$ and the excited $|333\rangle$ states leading to formation of their superposition with equal probability amplitudes as shown in Fig. 2b.

![Diagram](image_url)

**Fig. 1.** (a) The manifold of magnetic sublevels in ultracold $^{87}$Rb relevant for the studies of the electron dynamics, (b) A scheme of a three-atomic chain of Rydberg atoms coupled via van der Waals interactions.

**Fig. 2.** (a) Population of the ground and excited collective states as a function of one-photon detuning $\Delta$ for three-atomic system, $\Omega_0=1, V_{ij}=0.75 \omega_{21}$ (b) Population of states as a function of time leading to generation of the GHZ three-atomic state at the end of the pulse duration, $\Delta=-25.7 \omega_{21}$. 

The quantum control methodology to create the entangled spin states on demand is deduced from the dressed state picture and involves completely overlapping $\Omega_1$ and $\Omega_2$, and the interplay of the Rabi frequency with the one-photon detuning and the strength of the Rydberg-Rydberg interactions $V_{ij}$. The scheme implies the following control conditions: (i) The one-photon and two-photon detunings $\delta$ and $\Delta$ must be negative in sign so that they give the starting negative values of the collective state energies; (ii) Chirp rate $\alpha$ has to be negative so that the energy slope is positive and the energies of all collective states will cross the zero energy of the initial state $|111\rangle \omega_0=0$; (iii) The detuning must satisfy $|\Delta | > | \delta|$ so that the transitional states dependent on $\Delta$ are significantly shifted and do not resonate with $|111\rangle$ during the pulse duration; For the GHZ state, at the time when the crossings between the ground $|111\rangle$ and the $|333\rangle$ states occurs, the chirp of the pulse must be turned off to prevent further dynamics and population of the intermediate states.
With the methodology described above the three-atomic W state is readily produced as well. The superposition of the population of three contributing states $|331\rangle$, $|133\rangle$, and $|313\rangle$ - the W state - is plotted in Fig. 3 as a function of the one-photon detuning $\Delta$ and the Rabi frequency.

![Fig. 3. The W state as a function of the Rabi frequency and the one-photon detuning for the values of the Rydberg-Rydberg interaction $V_{21}=V_{32}=0.6$, $V_{23}=0.3$](image)

**Conclusion**

The methodology for generation of two basic classes of entangled states, the W and the GHZ states, is presented based on the two-photon adiabatic passage method, which makes use of two circularly polarized and linearly chirped pulses. The entangled three-atomic W and GHZ states are created by choosing the field parameters based on the developed quantum control scheme. This method aims for entangled state preparation in large ensembles.

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**References**